Research method "modeling on the basis of direct analogy"

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Rezumat: *în articol este descrisă metoda de cercetare „modelare în baza analogiei directe". Drept exemplu sunt prezentate analogii din electrotehnică, termotehnică, mecanică.*

Key words*: method, modeling, analogy, electrotechnics, termotechnics, mechanics.*

At present, researchers have at their disposal technical apparatus with the help of which it is possible to study physical processes in a new way, different from traditional methods. The new method is based on the modeling by direct analogy.

The essence of the new method consists in experimental research of physical process different in nature but developing in accordance with the same mathematical equations.

The method "modeling on the basis of direct analogy" differs from the method of physical modeling:

- "physical modeling" presupposes the use of physical models in which there are the same phenomena as in the object under research; a suitable example may be provided by research into flying apparatus in special aerodynamic pipes; the flight of an apparatus in real atmospheric conditions is substituted by the movement of a flying apparatus in the conditions similar to the real ones;

- "modeling on the basis of direct analogy" presupposes the use of physical modeling in which physical phenomena differ from those which occur in objects under research, but they can be described by the same mathematical equations; for example, to investigate the transfer of conductive warmth through metallic walls we use a physical model, consisting of electric components, described by a mathematical equation which has the same formula as a mathematical equation for thermic processes under research [7].

According to the nature of technical systems investigated with the help of the method "modeling on the basis of direct analogy" it is possible to distinguish electro-mechanic, electro- hydraulic, electro-acoustic analogies. Analogies connected with electric domain present special interest because physical models with electric components are produced relatively easily and their research is done without much difficulty.

Further, we present some points which will help to familiarize the pupils with the research method of modeling on the basis of direct analogy. There are good examples of electric models of conductive warmth transfer through homogeneous metallic models (electro-thermic analogy), and simple accelerometers (electro-mechanic analogy).

The familiarization of pupils with the method of "modeling on the basis of direct analogy" may be divided into the following stages:

- Defining a mathematical equation which approximately determines physical process under research;
- Defining models from other domains whose functioning can be studied easily and which are determined through mathematical equations of the same form in the same way as those equations that are researched.
- Defining magnitude analogy from the considered equations.

Electro-thermic analogy

1. Defining a mathematical equation which approximately determines the physical process under research (in our case, thermic process). Studying real technical systems, it is possible to state that their functioning can be described through mathematical equations, as a rule, approximately. Hence, the first step is to determinate the hypothesis which may lead to mathematical equations which interest us without distortion, at least, in some aspects, of the functioning of technical system under research. For example, studying the functioning of machines and thermic equipment with metallic walls whose breadth is small in comparison with their surface it is possible to consider that the conductive thermic flux which passes perpendicular to the walls surface is much bigger than the thermic flux which passes in the tangential direction on the surface. The hypothesis allows us to conclude that the conductive thermic flux passes through the metallic walls is unidirectional.

On the basis of this approximation, the conductive thermic unitary flux **q,** in other words the flux density of the warmth transmitted through the metallic walls [5], it will be expressed through the equation which is inferred from the Fourier low,

$$q = \frac{\lambda}{\sigma}(T_1 - T_2) \qquad (1)$$

where q – inductive unitary thermic flux, W/m^2**;**

λ – thermic conductivity in perpendicular direction to the surface of metallic walls, $\frac{W}{mK}$;

σ - breadth of walls, $m_{1,}$

T_1, T_2 – temperature on the sides of metallic walls, K.

The equation (1) can be written in the following form:

$$q=\frac{T_1-T_2}{\frac{\sigma}{\lambda}} \qquad (2).$$

2. Defining the physical model from another field (in our case, the field of electro-technics) whose functioning can be analyzed easily and which can expressed with help of the mathematical equation of the same form as the equation which refers to the process under research (in our case – thermic process).

In our case, such a physical model can be an electric circuit from the source of the permanent electric current with tension U, a consumer of electric energy with resistance R and a switch. On turning a switch, an electric current with intensity I will run through the circuit. The equation which describes the functioning of this physical model and which express the Ohm law [3,4] has the form

$$I=\frac{U}{R} \qquad (3)$$

where: I – electric current intensity, A;
U – electric tension, V;
R –electric resistance, Ω.

Assuming that for small currents, I depends on U, R is constant, the relation $\frac{\sigma}{\lambda}$ is constant, q depends on the difference between the temperatures T_1–T_2, it is possible to assert the equations (2) and (3) have the same mathematical form.

3. Defining analogical magnitudes from the considered equations (in our case – equation 2 and 3);

Comparing the equations (2) and (3) we state the following analogies:

- intensity I is an electric analogue of the conductive unitary thermic flux q;
- tension (difference of potential) U is an electric analogue of temperature difference (T_1-T_2).
- electric resistance R is an electric analogue of the relation $\frac{\sigma}{\lambda}$.

Electromechanic Analogy

1. Determining a mathematical equation which describes with some approximation the process under research (mechanic process). In principle, a simple accelerometer presents a slide bar situated horizontally, inside which there is a piston connected with it through the spring. Between the piston and a slide bar there is a lubricant. The piston serves an initial element which moves rectillineal to a slide bar.

If this mechanic system is fixed on a truck, then the accelerated movement of a truck horizontally by some exterior force F, the functioning of the accelerometer will be expressed through equation:

$$\frac{md^2x}{dt^2}+f\frac{dx}{dt}+rx=F_{(t)} \qquad (4),$$

where m – piston mass, kg;

x – abscissa horizontal to the piston in relation to a glide bar, m;

f – dynamic coefficient of viscosity, kg/s;

r – elasticity coefficient of the spring, N/m;

t – time, s;

$F_{(t)}$ – exterior force, N;

$\frac{md^2x}{dt^2}$ – inertness force of the piston; in accordance with Law II of Newton, inertness force is equal to the result between the mass and the speed [3,6];

$f\frac{dx}{dt}$ – viscous friction force is proportional to the speed [1];

rx – restraint force of a spring; in accordance with the law of Hooc, elasticity force is proportional to the lengthening of the body [3,6].

Taking into account the fact that:

$$\frac{d^2x}{dt^2} = \frac{dv}{dt}, \quad \frac{dx}{dt} = v, \qquad x = \int v dt$$

equation (4) can be written in the form:

$$\frac{mdv}{dt} + fv + r\int v\, dt = F_{(t)} \qquad (5).$$

2. Determining physical model from another domain (electro-technic domain), the functioning of which can be easily investigated and which can be described with the help of the mathematical equation of the same form as the equation which describes the process under research (in our case – mechanic process). In this case, an electric model of the accelerometer is an electric circuit of the type RLC consisting of electric energy source with variable tension U, a resistor with active electric resistance R, a coil with induction L, a condensator with capacity C.

The equation which describes the functioning of this electric system has the form:

$$\frac{LdI}{dt} + RI + \frac{1}{C}\int I\, dt = U \qquad (6)$$

where: L – induction, H;

I – electric current intensity, A;

t - time, s;

R - electric resistance, Ω;

C - capacity electric, F;

U – electric tension, V;

$\frac{LdI}{dt}$ - the falling of tension on the coil;

RI – fall of electric tension on active resistance ;

$\frac{1}{C}\int Idt$ – electric tension at condensator terminals.

As we see, the equations (5) and (6) have the same mathematical form.

3. *Determining analogical magnitude from the considered equations* (in our case – the equation (5) and (6)). The comparison of the equations (5) and (6) states the following analogies:

L→m;
I→v;
R→f;
$\frac{1}{C}$ →r;
U→F.

The practical value of the method (modeling on the basis of direct analogy) is that instead of studying the processes difficult to research by traditional methods we realize the corresponding analogue in the laboratory and with its help we can do experimental research.

All the facts considered show that University students and lycee students with technological studies and knowledge of the courses "Introduction into technology": with modules "Mechanics of solid bodies", "Hydraulics", "Thermotechnics", "Electrotechnics" [2] can be familiarized with the method without much difficulty.

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